TEST 3

(Math 140)

1. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$. (20 pts)

2. Find the volume of the parallelepid S determined by the vectors $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$. Also, find the area of the parallelogram determined by $\vec{u}$ and $\vec{v}$.

(20 pts)

3. Find the parametric equations of the line *l* that passes through P = (2,1,1) and is perpendicular to the plane *M* with equation $x - y = z$. (20 pts)

4. Identify the type of linear map the following represent: (20 pts)
(*Note*: By type, we mean whether is a reflection, projection or some other kind of linear map)

(a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$ (b) $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $C = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ (d) $D = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

5. Rotate the vector $\vec{u} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ around the vector $\vec{a} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$ by $270°$. (20 pts)

(*Note*: Use the following matrix: $A = \begin{pmatrix} a_1^2 + c(1-a_1^2) & a_1 a_2(1-c) - a_3 s & a_1 a_3(1-c) + a_2 s \\ a_1 a_2(1-c) + a_3 s & a_2^2 + c(1-a_2^2) & a_2 a_3(1-c) - a_1 s \\ a_1 a_3(1-c) - a_2 s & a_2 a_3(1-c) + a_1 s & a_3^2 + c(1-a_3^2) \end{pmatrix}$)