Key

TEST 3

(Math 140)

det 4

λ 4

Sytems 4

e_1 4

e_2 4

1. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$. (20 pts)

$\det \begin{vmatrix} -1-\lambda & 2 \\ 1 & 1-\lambda \end{vmatrix} = 0 \Rightarrow (-1-\lambda)(1-\lambda) - 2 = 0 \Rightarrow -1 + \lambda^2 = 2$

$\Rightarrow \lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$

So $Av = \lambda v$, Let $\lambda = -\sqrt{3}$

$\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\sqrt{3}\begin{bmatrix} x \\ y \end{bmatrix}$

① $\begin{cases} -x + 2y = -\sqrt{3}x \\ x + y = -\sqrt{3}y \end{cases}$ Now you can choose either, they will result in the same vector

② $x = (-\sqrt{3}-1)y \Rightarrow y = \dfrac{x}{(-\sqrt{3}-1)}$

$e_1 = \begin{bmatrix} x \\ \dfrac{x}{-\sqrt{3}-1} \end{bmatrix}$

Solving for e_2 $\begin{cases} -x + 2y = \sqrt{3}x \\ x + y = \sqrt{3}y \end{cases}$

$y = \dfrac{x}{\sqrt{3}-1} \cdot \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \dfrac{x(\sqrt{3}+1)}{2}$

$e_2 = \begin{bmatrix} x \\ \dfrac{x(\sqrt{3}+1)}{2} \end{bmatrix}$

2. Find the volume of the parallelepid S determined by the vectors $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$. Also, find the area of the parallelogram determined by $\vec{u}$ and $\vec{v}$.

(20 pts)

+10 $\vec{u} \times \vec{v}$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ -1 & 0 & 1 \end{vmatrix} \begin{matrix} i & j \\ 1 & 0 \\ -1 & 0 \end{matrix} = -2j - (1j) = -3j = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix}$

+5 $u \times v \cdot w$

$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{bmatrix} 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = |-9| = 9$

+5 pts. Mag.

$A(S) = \|\vec{u} \times \vec{v}\| = \sqrt{0^2 + (-3)^2 + 0^2} = 3$

or

+15 for Det.

$\vec{n}$ 8

eq. of Line 9

Final 3

3. Find the parametric equations of the line l that passes through P = (2,1,1) and is perpendicular to the plane M with equation $x - y = z$. (20 pts)

The normal to the plane, $\vec{n} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

The eq. of the line is:

$$\begin{aligned} x &= P_1 + \vec{n}_1 t \\ y &= P_2 + \vec{n}_2 t \\ z &= P_3 + \vec{n}_3 t \end{aligned} \Rightarrow \begin{aligned} x &= 2 + 1 \cdot t = 2 + t \\ y &= 1 + (-1)t = 1 - t \\ z &= 1 + (-1)t = 1 - t \end{aligned}$$

5 pts each

4. Identify the type of linear map the following represent: (20 pts)
(*Note*: By type, we mean whether is a reflection, projection or some other kind of linear map)

(a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$ Scaling

(b) $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ Projection

(c) $C = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ Rotation

(d) $D = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ Shearing

$\cos\alpha$ 2
$\sin\alpha$ 2
A 9
u' 7

5. Rotate the vector $\vec{u} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ around the vector $\vec{a} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$ by 270°. (20 pts)

$270° = \frac{3\pi}{2}$ $\cos\frac{3\pi}{2} = 0$ $\sin\frac{3\pi}{2} = -1$

(*Note*: Use the following matrix: $A = \begin{pmatrix} a_1^2 + c(1-a_1^2) & a_1a_2(1-c) - a_3s & a_1a_3(1-c) + a_2s \\ a_1a_2(1-c) + a_3s & a_2^2 + c(1-a_2^2) & a_2a_3(1-c) - a_1s \\ a_1a_3(1-c) - a_2s & a_2a_3(1-c) + a_1s & a_3^2 + c(1-a_3^2) \end{pmatrix}$)

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$$

$$\vec{u}' = A \cdot \vec{u} = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$